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ON THE OPTIMAL STABILIZATION OF THE POSITIONS OF A GYROSTAT-SATELLITE'S RELATIVE EQUILIBRIUM

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Using the method given in [1] the optimal stabilization of the positions of a satellite's relative equilibrium by means of flywheels, is studied. It is assumed that the center of mass of the gyrostat-satellite moves as a material point along a circular Keplerian orbit.

1. Let the center of mass of the gyrostat-satellite describe a circular orbit in a central Newtonian force field. We shall condsider a simplified problem, neglecting the influence of the motion about the mass center on the motion of the center itself.

We take the center of attraction o_1 as the origin of the inertial $o_1\xi\eta\zeta$-coordinate system, the mass center o of the satellite as the origin of the moving $ox_1x_2x_3$ - coordinate system and we direct the axes along the principal central axes of inertia. We introduce another moving $oxyz$-coordinate system the z-axis of which is directed along the line o_1o, the y-axis along the normal to the plane stationary circular orbit and the x-axis complementing the y- and z-axes to a right trihedron. We define the position of the body of the satellite in the orbital $oxyz$-coordinate system in terms of the Euler angles ψ, θ, φ. We denote α_i, β_i, γ_i $(i = 1, 2, 3)$ the cosines of the angles between the x, y, z and x_1, x_2, x_3 axes, and define them as follows:

$$\cos(z, x_i) = \alpha_i, \quad \cos(y, x_i) = \beta_i, \quad \cos(x, x_i) = \gamma_i \quad (i = 1, 2, 3)$$
$$\alpha_1 = \sin\varphi\sin\theta, \quad \alpha_2 = \cos\varphi\sin\theta, \quad \alpha_3 = \cos\theta$$
$$\beta_1 = \cos\varphi\sin\psi + \sin\varphi\cos\psi\cos\theta, \quad \beta_2 = -\sin\varphi\sin\psi + \cos\varphi\cos\psi\cos\theta, \quad \beta_3 = -\sin\theta\cos\psi$$
$$\gamma_1 = \alpha_3\beta_2 - \alpha_2\beta_3, \quad \gamma_2 = \alpha_1\beta_3 - \alpha_3\beta_1, \quad \gamma_3 = \alpha_2\beta_1 - \alpha_1\beta_2$$

Let the axes of the three homogeneous symmetric flywheels be directed along the principal axes of inertia of the satellite and $\omega(p, q, r)$ denote the angular velocity of rotation of the satellite about the center of mass, and let p, q, r be the projections of the angular velocity on the axes of the moving $ox_1x_2x_3$ - coordinate system. We assume that the force function of Newtonian attraction of the satellite has the form [2]

$$U(\alpha_1, \alpha_2, \alpha_3) = \mu M R_0^{-1} - 3\mu(2R_0^3)^{-1}[C_1\alpha_1^2 + C_2\alpha_2^2 + C_3\alpha_3^2 - {}^1/_3(C_1 + C_2 + C_3)]$$

where M is the mass of the gyrostat-satellite, C_i are its principal moments of inertia and μ is the gravitational constant.

The equations of absolute motion of the satellite about its center of mass can be written in the form of three dynamic Euler equations

$$C_1p^{\cdot} + (C_3 - C_2)\, qr + H_3q - H_2r + H_1^{\cdot} = 3\omega_0^2\,(C_3 - C_2)\alpha_2\alpha_3 \qquad (1\ 2\ 3) \qquad (1.1)$$

$$(H_i = J_i\omega_i,\ i = 1, 2, 3)$$

Here J_i and ω_i are the axial moments of inertia and relative angular velocities of the flywheels and ω_0 is the angular velocity of motion of the center of mass along the orbit. The symbol $(1\ 2\ 3)$ indicates that the remaining two equations can be obtained by cyclic permutation. The equations determining the position of the satellite in the orbital $oxyz$ -coordinate system have the form

$$\alpha_1^{\bullet} = \alpha_2 r - \alpha_3 q + \omega_0\,(\alpha_3\beta_2 - \alpha_2\beta_3) \qquad (1\ 2\ 3)$$
$$\beta_1^{\bullet} = \beta_2 r - \beta_3 q \qquad (1\ 2\ 3) \qquad (1.2)$$

In addition to Eqs. (1.1) and (1.2), we shall consider another three equations describing the rotational motions of the flywheels. With the internal friction neglected, the equations have the form

$$J_1\,(\omega_1^{\bullet} + p^{\bullet}) = -u_1,\ J_2\,(\omega_2^{\bullet} + q^{\bullet}) = -u_2,\ J_3\,(\omega_3^{\bullet} + r^{\bullet}) = -u_3 \qquad (1.3)$$

where u_i are the moments of the motors responsible for the rotation of the flywheels.

From (1.1) and (1.3) we obtain

$$(C_1 - J_1)\,p^{\bullet} = (C_2q + H_2)r - (C_3r + H_3)\,q + 3\omega_0^2\,(C_3 - C_2)\times \alpha_2\alpha_3 + u_1 \qquad (1\ 2\ 3) \qquad (1.4)$$

When $u_i = 0$ $(i = 1, 2, 3)$, the equations of motion of the flywheels (1.3) have the following integrals:

$$H_1 + J_1p = l_1,\ H_2 + J_2q = l_2,\ H_3 + J_3r = l_3,\quad l_i = \text{const} \qquad (i = 1, 2, 3) \qquad (1.5)$$

When the center of mass moves uniformly along the circular orbit $(u_i = 0)$, the equations of motion (1.4) and (1.2) admit the energy integral [3] which, with (1.5) taken into account, has the form

$$2H = (C_1 - J_1)\,p^2 + (C_2 - J_2)\,q^2 + (C_3 - J_3)\,r^2 + 3\omega_0^2\,(C_1\alpha_1^2 + C_2\alpha_2^2 + C_3\alpha_3^2) - 2\omega_0\,[(C_1 - J_1)\,p\beta_1 + (C_2 - J_2)\,q\beta_2 + (C_3 - J_3)\,r\beta_3] - 2\omega_0\,(l_1\beta_1 + l_2\beta_2 + l_3\beta_3) \qquad (1.6)$$

The set of positions of relative equilibrium was fully determined in [4] under the assumption that the flywheels rotate with constant relative angular velocities

($u_i = 0,\ i = 1, 2, 3$), while in [5] it was shown that all positions of the relative equilibrium of the gyrostat-satellite can be divided into three classes.

1.1. One of the principal axes of the inertia ellipsoid of the satellite is collinear with the z-axis, and the other two axes are located in the oxy-plane forming a certain angle with the x- and y-axes.

1.2. One of the principal axes of the inertia ellipsoid of the satellite is collinear with the x-axis and the other two axes lie in the oyz-plane forming a certain angle with the y- and z-axes.

1.3. None of the principal axes of inertia of the satellite are collinear with the axes of the orbital coordinate system.

Let us consider one of the positions of relative equilibrium belonging to class 1.1. Let e.g. the x_2-axis be collinear with the z-axis and let the x_1, x_3 -axes lie in the oxy -plane forming the angle ψ_0 with the x- and y -axes. Let also

$$\theta_0 = {}^1/_2\pi, \quad 0 \leqslant \psi_0 \leqslant 2\pi, \quad \varphi_0 = 0$$

We have

$$\begin{aligned} &p = \omega_0 \sin \psi_0, \quad q = 0, \quad z = -\omega_0 \cos \psi_0 \\ &\alpha_1 = 0, \quad \alpha_2 = 1, \quad \alpha_3 = 0 \\ &\beta_1 = \sin \psi_0, \quad \beta_2 = 0, \quad \beta_3 = -\cos \psi_0 \end{aligned} \qquad (1.7)$$

under the condition that

$$H_2 = 0, \quad \omega_0 (C_3 - C_1) \sin \psi_0 \cos \psi_0 = H_1 \cos \psi_0 + H_3 \sin \psi_0$$

Let us write the equations of perturbed motion, adopting the following notation for the variations in the variables:

$$\begin{aligned} &p_1 = p - \omega_0 \sin \psi_0, \quad \eta_1 = \alpha_1, \quad \delta_1 = \beta_1 - \sin \psi_0 \\ &q_1 = q, \quad \eta_2 = \alpha_2 - 1, \quad \delta_2 = \beta_2 \\ &r_1 = r + \omega_0 \cos \psi_0, \quad \eta_3 = \alpha_3, \quad \delta_3 = \beta_3 + \cos \psi_0 \end{aligned} \qquad (1.8)$$

We have

$$\begin{aligned} &(C_1 - J_1)\, p_1^{\cdot} = (C_2 - C_3) q_1 r_1 - (C_2 - C_3)\, \omega_0 q_1 \cos \psi_0 - H_3 q_1 + \\ &\quad 3\omega_0^2 (C_3 - C_2)(1 + \eta_2)\, \eta_3 + u_1 \quad (1\ 2\ 3) \\ &\eta_1^{\cdot} = \eta_2 r_1 - \eta_3 q_1 + r_1 - \omega_0 \delta_3 + \omega_0 (\eta_3 \delta_2 - \eta_2 \delta_3) \quad (1\ 2\ 3) \\ &\delta_1^{\cdot} = \delta_2 r_1 - \delta_3 q_1 + q_1 \cos \psi_0 - \omega_0 \delta_2 \cos \psi_0 \quad (1\ 2\ 3) \end{aligned} \qquad (1.9)$$

The variables η_i and δ_i satisfy the relations

$$\begin{aligned} &\Phi_1 = \delta_1^2 + \delta_2^2 + \delta_3^2 + 2\delta_1 \sin \psi_0 - 2\delta_3 \cos \psi_0 = 0 \\ &\Phi_2 = \eta_1^2 + \eta_2^2 + \eta_3^2 + 2\eta_2 = 0 \\ &\Phi_3 = \delta_1 \eta_1 + \delta_2 \eta_2 + \delta_3 \eta_3 + \eta_1 \sin \psi_0 + \delta_2 - \eta_3 \cos \psi_0 = 0 \end{aligned} \qquad (1.10)$$

Having written the integral (1.6) in terms of the variables (1.8), we consider the relation connecting the functions (1.6) and (1.10) in the form

$$2V = 2H + \lambda\omega_0\Phi_1 - 3\omega_0^2C_2\Phi_2 = (C_1 - J_1)\,p_1^2 + (C_2 - J_2)q_1^2 + (C_3 - J_3)\,r_1^2 - 2\omega_0\,[(C_1 - J_1)\,p_1\delta_1 + (C_2 - J_2)\,q_1\delta_2 + (C_3 - J_3)\,r_1\delta_3] + \lambda\omega_0\,(\delta_1^2 + \delta_2^2 + \delta_3^2) + 3\omega_0^2\,[(C_1 - C_2)\,\eta_1^2 + (C_3 - C_2)\eta_3^2] \qquad (1.11)$$

where $\lambda = \text{const} > 0$, and

$$l_1 = \lambda \sin'\psi_0 - (C_1 - J_1)\,\omega_0 \sin\psi_0,\ l_2 = 0,\ l_3 = -\lambda\cos\psi_0 + (C_3 - J_3)\,\omega_0\cos\psi_0 \qquad (1.12)$$

in the expression for $2H$. When

$$\lambda > \max\,\{\omega_0 C_1,\ \omega_0 C_3\},\quad C_1 > C_2,\ C_3 > C_2 \qquad (1.13)$$

the function (1.11) is a positive definite function of the variables p_1, q_1, r_1, δ_1, δ_2, δ_3, η_1, and η_3. From (1.12), (1.5) and (1.7) we find λ

$$\lambda = \frac{H_1}{\sin\psi_0} + \omega_0C_1,\quad \lambda = -\frac{H_3}{\cos\psi_0} + \omega_0C_3 \qquad (1.14)$$

Let us assume that

$$C_1 \neq C_3,\quad C_1 > C_2,\quad C_3 > C_2 \qquad (1.15)$$

Then the condition (1.13) becomes, with (1.14) amd (1.15) taken into account,

$$\omega_0C_1 + \frac{H_1}{\sin\psi_0} > \max\,\{\omega_0C_1,\ \omega_0C_3\}$$
$$\omega_0C_3 - \frac{H_3}{\cos\psi_0} > \max\,\{\omega_0C_1,\ \omega_0C_3\} \qquad (1.16)$$

Following [1]. we consider the functional

$$T = \int_{t_0}^{\infty}(F\,(p_1, q_1, r_1, \delta_1, \delta_2, \delta_3) + \sum_{i=1}^{3} n_iu_i^2)\,dt \qquad (1.17)$$

where F is a nonnegative function to be determined and n_i are some positive numbers. Let us set the following expression [6];

$$B\,[V; p_1, q_1, r_1, \delta_1, \delta_2, \delta_3; u_1, u_2, u_3] = (p_1 - \omega_0\delta_1)\,u_1 + (q_1 - \omega_0\delta_2)\,u_2 + (r_1 - \omega_0\delta_3)\,u_3 + F + \sum_{i=1}^{3} n_iu_i^2 \qquad (1.18)$$

which, in accordance with the theory of optimal stabilization, reaches a minimum equal to zero at $u_i = u_i^0$. The optimal controls u_i are found from the equations

$$\partial B / \partial u_j^0 = 0 \quad (j = 1, 2, 3)$$

and have the form

$$u_1^\circ = -{}^1/_2 n_1 (p_1 - \omega_0 \delta_1), \quad u_2^\circ = -{}^1/_2 n_2 (q_1 - \omega_0 \delta_2), \\ u_3^\circ = -{}^1/_2 n_3 (r_1 - \omega_0 \delta_3) \tag{1.19}$$

Substituting the expressions for u_i from (1.19) into (1.18) and equating the resulting expression to zero [1], we obtain the function

$$F = {}^1/_4 [{}^1/n_1 (p_1 - \omega_0 \delta_1)^2 + {}^1/n_2 (q_1 - \omega_0 \delta_2)^2 + {}^1/n_3 \times \\ (r_1 - \omega \delta_3)^2] \tag{1.20}$$

The time derivative of (1.11) is, by virtue of the system of equations of perturbed motion (1.9) with (1.19), (1.17) and (1.20) taken into account

$$V^\cdot = -2F \tag{1.21}$$

The function (1.21) is a negative sign-constant of the variables p_1, q_1, r_1, δ_i, and η_i and the manifold E of points at which $V^\cdot = 0$ has the form

$$p_1 = \omega_0 \delta_1, \quad q_1 = \omega_0 \delta_2, \quad r_1 = \omega_0 \delta_3, \quad \eta_i - \text{ are arbitrary} \tag{1.22}$$

We shall show that in a sufficiently small neighborhood of the unperturbed motion

$$p_1 = q_1 = r_1 = 0, \quad |\delta_i = 0, \quad \eta_i = 0 \quad (i = 1, 2, 3) \tag{1.23}$$

the manifold (1.22) contains none other than the unperturbed motion (1.23).

The equations of motions (1.9) assume, for the values given by (1.22), the following form:

$$[\omega_0^2 (C_2 - C_3)(\delta_3 - \cos\psi_0) - \omega_0 H_3]\delta_2 = 3\omega_0^2 (C_2 - C_3) \cdot \\ (1 + \eta_2)\eta_3 \\ [\omega_0^2 (C_1 - C_2)(\delta_1 + \sin\psi_0) + \omega_0 H_1]\delta_2 = 3\omega_0^2 (C_1 - C_2) \cdot \\ (1 + \eta_2)\eta_1 \\ \omega_0 (C_3 - C_1)(\delta_1\delta_3 + \delta_3 \sin\psi_0 - \delta_1 \cos\psi_0) + H_3\delta_1 - H_1\delta_3 = \\ 3\omega_0 (C_3 - C_1)\eta_1\eta_3 \tag{1.24}$$

Let the values of δ_i and η_i $(i = 1, 2, 3)$ exist satisfying the system (1.24). Substituting the values of δ_i into (1.24) we can obtain equations which will yield η_i

$$3\omega_0^2 (C_2 - C_3)(1 + \eta_2)\eta_3 = a_1, \quad 3\omega_0^2 (C_3 - C_1)\eta_1\eta_3 = a_2 \\ 3\omega_0^2 (C_1 - C_2)(1 + \eta_2)\eta_1 = a_3, \quad a_i = \text{const} \quad (i = 1, 2, 3) \tag{1.25}$$

If $a_i \neq \text{const}$ $(i = 1, 2, 3)$ then a region

$$\eta_1^2 + \eta_2^2 + \eta_3^2 < m^2 = \text{const} \tag{1.26}$$

can always be found in which the system (1.25) has no solutions. In fact, multiplying the equations of the system (1.25) by η_1, η_2, and η_3 respectively, we obtain

$$a_1\eta_1 + a_2\eta_2 + a_3\eta_3 = -a_2 \tag{1.27}$$

If we take the distance between the point $\eta_i = 0$ $(i = 1, 2, 3)$ and the plane (1.27) as m, the system (1.25) will have no solution in the region (1.26) when $a_2 \neq 0$. We note that when $a_2 = 0$, the parameter λ in (1.11) can be chosen such that a_1 and a_3 will also vanish. Consequently, the necessary condition for the system (1.24) to have a solution in some region of the unperturbed motion (1.23) is that $a_i = 0$ $(i = 1, 2, 3)$ Let $a_i = 0$ $(i = 1, 2, 3)$. Then the system (1.24) separates into two independent systems

$$(1 + \eta_2)\eta_3 = 0, \quad (1 + \eta_2)\eta_1 = 0, \quad \eta_1\eta_3 = 0 \tag{1.28}$$

$$\delta_2[\omega_0(C_2 - C_3)\delta_3 - \omega_0(C_2 - C_3)\cos\psi_0 - H_3] = 0 \tag{1.29}$$

$$\delta_2[\omega_0(C_1 - C_2)\delta_1 + \omega_0(C_1 - C_2)\sin\psi_0 + H_1] = 0$$

$$\omega_0(C_3 - C_1)(\delta_1\delta_3 + \delta_3\sin\psi_0 - \delta_1\cos\psi_0) + (H_3\delta_1 - H_1\delta_3) = 0$$

Equations (1.28) together with $\Phi_2 = 0$ from (1.10), have a unique solution $\eta_1 = \eta_2 = \eta_3 = 0$ in the region

$$\eta_1^2 + \eta_2^2 + \eta_3^2 < 2 \tag{1.30}$$

The first two equations of (1.29) vanish when $\delta_2 = 0$, or when

$$\delta_3 = \cos\psi_0 + \frac{H_3}{\omega_0(C_2 - C_3)} \quad \text{и} \quad \delta_1 = -\sin\psi_0 - \frac{H_1}{\omega_0(C_1 - C_2)} \tag{1.31}$$

If however the parameter λ is chosen in accordance with the inequality

$$\frac{(\lambda - \omega_0 C_1)^2 \sin^2\psi_0}{\omega_0^2(C_1 - C_2)^2} + \frac{(\lambda - \omega_0 C_3)^2\cos^2\psi_0}{\omega_0^2(C_2 - C_3)^2} > 1 \tag{1.32}$$

then the equation

$$\delta_1^2 + \delta_2^2 + \delta_3^2 + 2\delta_1\sin\psi_0 - 2\delta_3\cos\psi_0 = 0 \tag{1.33}$$

has no real solution in δ_2 for the values given by (1.31). Therefore, when (1.32) holds, the first two equations of (1.29) are satisfied only when $\delta_2 = 0$.

Let us consider the third equation of (1.29) together with (1.33). Substituting the expression for δ_1 from the third equation of (1.29) into (1.33), we have

$$f(\delta_3) = \delta_3\varphi(\delta_3) = \delta_3\left[\frac{k_2^2\delta_3}{(k_1+\delta_3)^2} + 2\frac{k_2\sin\psi_0}{k_1+\delta_3} + \delta_3 - 2\cos\psi_0\right] = 0$$

$$k_1 = -\frac{\lambda - \omega_0 C_1}{\omega_0(C_3 - C_1)}\cos\psi_0, \quad k_2 = \frac{\lambda - \omega_0 C_3}{\omega_0(C_3 - C_1)}\sin\psi_0$$

The function $\varphi(\delta_3)$ changes its sign on the segment $[-1 + \cos\psi_0, 1 + \cos\psi_0]$ only once. If we denote the root of the equation $\varphi(\delta_3) = 0$ by δ_{30}, the system (1.29) with (1.32) will obviously have a unique solution $\delta_1 = \delta_2 = \delta_3 = 0$ in the region

$$\delta_1^2 + \delta_2^2 + \delta_3^2 < \varepsilon^2, \quad \varepsilon^2 = \delta_{10}^2 + \delta_{30}^2 \tag{1.34}$$

Thus when (1.15), (1.16) and (1.32) hold, the controls (1.19) ensure that asymptotic stability [7] of the unperturbed motion (1.23) for all initial perturbations belonging to the region (1.30), (1.34), and minimize the functional

$$J = \frac{1}{4}\int_{t_0}^{\infty}\Big[\frac{1}{n_1}(p_1 - \omega_0\delta_1)^2 + \frac{1}{n_2}(q_1 - \omega_0\delta_2)^2 + \frac{1}{n_3}(r_1 - \omega_0\delta_3)^2 + 4\sum_{i=1}^{3} n_i u_i^2\Big] dt \qquad (1.35)$$

2. Consider any position of relative equilibrium belonging to the class 1.2. Let e.g. the x_1-axis be collinear with the x-axis, the x_2- and x_3-axes lie in the oyz-plane and form an angle θ_0 with the y- and z-axes, and

$$\psi_0 = \pi, \quad 0 < \theta_0 < \pi, \quad \varphi = \pi$$

We have

$$p = 0, \quad q = \omega_0 \cos\theta_0, \quad r = \omega_0 \sin\theta_2$$
$$\alpha_1 = 0, \quad \alpha_2 = -\sin\theta_0, \quad \alpha_3 = \cos\theta_0$$
$$\beta_1 = 0, \quad \beta_2 = \cos\theta_0, \quad \beta_3 = \sin\theta_0$$

under the condition that

$$H_1 = 0, \quad 4\omega_0(C_2 - C_3)\sin\theta_0\cos\theta_0 = H_3\cos\theta_0 - H_2\sin\theta_0$$

Retaining the notation (1.8) for the variations of the variables, we set the following equations of perturbed motion:

$$(C_1 - J_1)\, p_1^{\cdot} = (C_2 q_1 + H_2 + \omega_0 C_2\cos\theta_0)(r_1 + \omega_0\sin\theta_0) - (C_3 r_1 + H_3 + \omega_0 C_3 \sin\theta_0)(q_1 + \omega_\theta\cos\theta_0) + 3\omega_0^2 (C_3 - C_2)(\eta_2 - \sin\theta_0)(\eta_3 + \cos\theta_0) + u_1 \qquad (1\ 2\ 3) \qquad (2.1)$$

$$\eta_1^{\cdot} = \eta_2 r_1 - \eta_3 q_1 + \omega_0(\eta_3\delta_2 - \eta_2\delta_3) + r_1\sin\theta_0 - q_1\cos\theta_0 + \omega_0(\delta_2\cos\theta_0 + \delta_3\sin\theta_0) \qquad (1\ 2\ 3)$$

$$\delta_1^{\cdot} = \delta_2 r_1 - \delta_3 q_1 + \omega_0(\delta_2\sin\theta_0 - \delta_3\cos\theta_0) + r_1\cos\theta_0 - q_1\sin\theta_2 \qquad (1\ 2\ 3)$$

The relations (1.10) now become

$$\Phi_1 = \delta_1^2 + \delta_2^2 + \delta_3^2 + 2\delta_2\cos\theta_0 + 2\delta_3\sin\theta_0 = 0 \qquad (2.2)$$
$$\Phi_2 = \eta_1^2 + \eta_2^2 + \eta_3^2 + 2\eta_3\cos\theta_0 - 2\eta_2\sin\theta_0 = 0$$
$$\Phi_3 = \delta_1\eta_1 + \delta_2\eta_2 + \delta_3\eta_3 + \eta_2\cos\theta_0 - \delta_2\sin\theta_0 + \eta_3\sin\theta_0 + \delta_3\cos\theta_2 = 0$$

Let the following condition hold for the mements of inertia of the gyrostat-satellite:

$$C_1 > C_2 = C_3 \qquad (2.3)$$

Having written the integral (1.6) in terms of variations of the variables, we consider the relation connecting the functions (1.6) and (2.2), of the form

$$2V = 2H + \lambda_1\omega_0\Phi_1 - 3\omega_0^2C_3\Phi_2 = (C_1 - J_1)\,p_1^2 + (C_2 - J_2)\,q_1^2 + (C_3 - J_3)r_1^2 - 2\omega_0\,[(C_1 - J_1)\,p_1\delta_1 + (C_2 - J_2)\,q_1\delta_2 + (C_3 - J_3)r_1\delta_3] + \lambda_1\omega_0\,(\delta_1^2 + \delta_2^2 + \delta_3^2) + 3\omega_0^2\,(C_1 - C_2)\,\eta_1^2 \tag{2.4}$$

where $\lambda_1 = \text{const} > 0$ and where in the expression for $2H$ we have set

$$l_1 = 0,\quad l_2 + \omega_0\,(C_2 - J_2)\cos\theta_0 = \lambda_1\cos\theta_0,\quad l_3 + \omega_0\cdot(C_3 - J_3)\sin\theta_0 = \lambda_1\sin\theta_0 \tag{2.5}$$

When (2.3) and

$$\lambda_1 > \omega_0 C_1 \tag{2.6}$$

the function (2.4) is a positive definite function of the variables p_1, q_1, r_1, δ_1, δ_2, δ_3 and η_1.

Repeating the above arguments, we can find the optimal controls u_i°. The controls have the form (1.19), the function F in the expression (1.17) has the form (1.20), and the time derivative of the function (2.4)

$$2V^{\cdot} = -[{}^1/n_1\,(p_1 - \omega_0\delta_1)^2 + {}^1/\,n_2\,(q_1 - \omega_0\delta_2)^2 + {}^1/\,n_3\cdot\cdot(r_1 - \omega_0\delta_3)^2]$$

constructed using the equations of perturbed motion (2.1) with (1.19), (1.17) and (1.20) taken into account, is a negative sign-constant function of variations of the variables used.

Similarly we can show that when the conditions (2.3), (2.6) and

$$\frac{\lambda_1 - \omega_0 C_1}{\omega_0\,(C_1 - C_2)}\,|\cos\theta_0| > 2 \tag{2.7}$$

all hold, the manifold (1.22) in which $V^{\cdot} = 0$ contains no complete motions of the system in the region

$$\delta_1^2 + \delta_2^2 + \delta_3^2 < 4,\quad \eta_1^2 + \eta_2^2 + \eta_3^2 < 2 \tag{2.8}$$

except the unperturbed motion (1.23). Thus, when the conditions (2.3), (2.6) and (2.7) all hold, the controls (1.19) ensure the asymptotic stability of the unperturbed motion (1.23) relative to the variables p_1, q_1, r_1, δ_1, δ_2, δ_3 and η_1 for all initial perturbations belonging to the region (2.8), and minimize the functional (1.35).

In [8] it was shown that when $C_1 \neq C_2 \neq C_3$ and $\theta_0 \neq {}^1/_4\pi$ and ${}^3/_4\pi$, all positions of the relative equilibrium of class 1.2 can be asymptotically stabilized by moments applied to the flywheels. If on the other hand either the inertia ellipsoid of the gyrostat-satellite is an ellipsoid of revolution i.e. $C_1 > C_2 = C_3$ or $\theta_0 = {}^1/_4\pi$, ${}^3/_4\pi$, then the position of relative equilibrium cannot be stabilized with respect to all variables by applying moments to the flywheels since in these cases the system is not fully controllable [9]. Nevertheless, as it was shown before, in these cases we can still attain the asymptotic stability of the positions of relative equilibrium with respect to a part of the variables [1] by applying moments to the flywheels.

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